Mathematics — Solved Paper 2017

SECTION A (40 Marks)

*(Answer **all** questions from this Section)*

***Question* 1 :**

(a) If b is the mean proportion between a and c, show that :

$$\frac{a^4+a^2b^2+b^4}{b^4+b^2c^2+c^4} = \frac{a^2}{c^2}. \qquad \textbf{[3]}$$

Solution :

b is the mean proportion between a and c $\Rightarrow$ $b^2 = ac$

$\Rightarrow \quad b^4 = (b^2)^2 = (ac)^2 = a^2c^2$

$$\therefore \quad \textbf{L.H.S.} = \frac{a^4+a^2b^2+a^2c^2}{a^2c^2+b^2c^2+c^4} \qquad [\because b^4 = a^2c^2]$$

$$= \frac{a^2(a^2+b^2+c^2)}{c^2(a^2+b^2+c^2)} = \frac{a^2}{c^2} = \textbf{R.H.S.}$$

Hence Proved.

(b) Solve the equation $4x^2 - 5x - 3 = 0$ and give your answer correct to two decimal places. [4]

Solution :

Comparing $4x^2 - 5x - 3 = 0$ with $ax^2 + bx + c = 0$; we get :

$a = 4$, $b = -5$ and $c = -3$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25 - 4 \times 4 \times -3}}{2 \times 4}$$

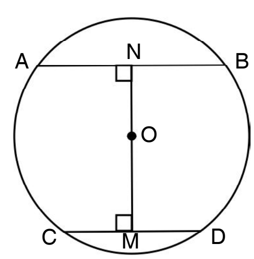
$$= \frac{5 \pm \sqrt{25+48}}{8}$$

$$= \frac{5 \pm \sqrt{73}}{8}$$

$$= \frac{5 \pm 8{\cdot}544}{8} = 1{\cdot}693 \text{ and } -0{\cdot}443 = \mathbf{1{\cdot}69 \text{ and } -0{\cdot}44}$$

Ans.

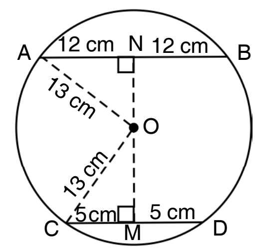
(c) AB and CD are two parallel chords of a circle such that AB = 24 cm and CD = 10 cm. If the radius of the circle is 13 cm, find the distance between the two chords. [3]



Solution :

$\because$ Perpendicular from the centre of a circle bisects the chords.

$\therefore$ AN = BN = $\frac{1}{2}$ × AB = $\frac{1}{2}$ × 24 cm = 12 cm



And, CM = DM = $\frac{1}{2}$ × CD = 5 cm

Given radius of the circle is 13 cm

$\Rightarrow$ OA = OC = 13 cm

In right triangle OAN,

$ON^2 + AN^2 = OA^2$

$\Rightarrow$ $ON^2 + 12^2 = 13^2$ $\Rightarrow$ ON = 5 cm

In right triangle OCM,

$OM^2 + CM^2 = OC^2$

$\Rightarrow$ $OM^2 + 5^2 = 13^2$ $\Rightarrow$ OM = 12 cm

$\therefore$ **The distance between the two chords**

= MN

= ON + OM = 5 cm + 12 cm = **17 cm** **Ans.**

***Question* 2 :**

(a) Evaluate without using trigonometric tables,

$$\mathbf{\sin^2 28^\circ + \sin^2 62^\circ + \tan^2 38^\circ - \cot^2 52^\circ + \frac{1}{4}\sec^2 30^\circ.}$$ **[3]**

Solution :

$$= \sin^2 28^\circ + \sin^2(90^\circ - 28^\circ) + \tan^2 38^\circ - \cot^2(90^\circ - 38^\circ) + \frac{1}{4} \times \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= \sin^2 28^\circ + \cos^2 28^\circ + \tan^2 38^\circ - \tan^2 38^\circ + \frac{1}{3}$$

$$= 1 + 0 + \frac{1}{3} = \mathbf{1\frac{1}{3}}$$ **Ans.**

(b) If A = $\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ and $A^2 - 5B^2 = 5C$. Find matrix C where C is a 2 by 2 matrix. **[4]**

Solution :

$\because$ $$A^2 = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1+9 & 3+12 \\ 3+12 & 9+16 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 - 5B^2 = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 10-5 & 15-0 \\ 15-0 & 25-5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix}$$

$$\Rightarrow \quad 5C = A^2 - 5B^2 \Rightarrow 5C = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} \Rightarrow \mathbf{C} = \begin{bmatrix} \mathbf{1} & \mathbf{3} \\ \mathbf{3} & \mathbf{4} \end{bmatrix} \qquad \textbf{Ans.}$$

(c) Jaya borrowed ₹ 50,000 for 2 years. The rates of interest for two successive years are 12% and 15% respectively. She repays ₹ 33,000 at the end of the first year. Find the amount she must pay at the end of the second year to clear her debt. [Not in ICSE course now] [3]

***Question* 3 :**

(a) The catalogue price of a computer set is ₹ 42,000. The shopkeeper gives a discount of 10% on the listed price. He further gives an off-season discount of 5% on the discounted price. However, sales tax at 8% is charged on the remaining price after the two successive discounts. Find :

(i) the amount of sales tax a customer has to pay.

(ii) the total price to be paid by the customer for the computer set. [3]

Solution :

The catalogue (list) price of the computer = ₹ 42,000

First discount given by the shopkeeper = 10% of ₹ 42,000 = ₹ 4,200

And, ₹ 42,000 – ₹ 4,200 = ₹ 37,800

Further off-season discount = 5% of ₹ 37,800 = ₹ 1,890

Remaining price after the two successive discounts

= ₹ 37,800 – ₹ 1,890 = ₹ 35,910

(i) **Amount of sales tax a customer has to pay**

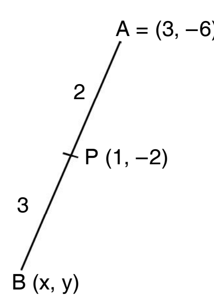
= 8% of ₹ 35,910 = **₹ 2,872.80** **Ans.**

(ii) **The total price to be paid by the customer for the computer set**

= ₹ 35,910 + ₹ 2,872.80 = **₹ 38,782.80** **Ans.**

(b) P(1, –2) is a point on the line segment A(3, –6) and B(*x, y*) such that AP : PB is equal to 2 : 3. Find the coordinates of B. [4]

Solution :



$$P(x) = \frac{2 \times x + 3 \times 3}{2+3} \Rightarrow 1 = \frac{2x+9}{5}$$

i.e. $5 = 2x + 9$ and $x = -2$

$$P(y) = \frac{2 \times y + 3 \times -6}{2+3} \Rightarrow -2 = \frac{2y-18}{5}$$

i.e. $-10 = 2y - 18$ and $y = 4$

$\therefore$ **Coordinates of B = (–2, 4)** **Ans.**

(c) The marks of 10 students of a class in an examination arranged in ascending order is as follows :

13, 35, 43, 46, x, x + 4, 55, 61, 71, 80

If the median marks is 48, find the value of x. Hence, find the mode of the given data. [3]

Solution :

The median marks is 48 $\Rightarrow \dfrac{x+x+4}{2} = 48$

$\Rightarrow 2x + 4 = 96$ and $\boldsymbol{x = 46}$ **Ans.**

$\because$ The marks of 10 students are

= 13, 35, 43, 46, 46, 50, 55, 61, 71 and 80

Clearly, **Mode = 46** **Ans.**

***Question* 4 :**

(a) What must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor ? [3]

Solution :

Let y must be subtracted.

Clearly, $16x^3 - 8x^2 + 4x + 7 - y$ has $2x + 1$ as a factor

$$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$$

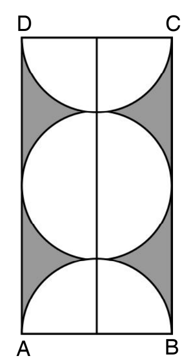
$$\therefore \quad 16\left(-\frac{1}{2}\right)^3 \times - 8\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 7 - y = 0$$

$$\Rightarrow \quad 16 \times -\frac{1}{8} - 8 \times \frac{1}{4} - 4 \times \frac{1}{2} + 7 - y = 0$$

$$\Rightarrow \quad -2 - 2 - 2 + 7 - y = 0$$

i.e. $\boldsymbol{y = 1}$ **Ans.**

(b) In the given figure ABCD is a rectangle. It consists of a circle and two semi circles each of which are of radius 5 cm. Find the area of the shaded region. Give your answer correct to three significant figures. [4]



Solution :

Since, radius of the circle and of each semicircle is 5 cm, width (AB) of the rectangle ABCD = 2 × r = 2 × 5 cm = 10 cm and height (AD) of the rectangle ABCD = 4 × r = 4 × 5 cm = 20 cm

$\therefore$ **Area of the shaded portion**

= Area of rectangle ABCD – Area of one circle – sum of the areas of two semi circles

$= 10 \text{ cm} \times 20 \text{ cm} - \pi(5)^2 - \dfrac{\pi(5)^2}{2} \times 2$

$= 200 \text{ cm}^2 - 78{\cdot}6 \text{ cm}^2 - 78{\cdot}6 \text{ cm}^2 = \mathbf{42{\cdot}8 \text{ cm}^2}$ **Ans.**

(c) Solve the following inequation and represent the solution set on a number line.

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \le 7\frac{1}{2},\ x \in I$$ **[3]**

Solution :

$$-8\frac{1}{2} < -\frac{1}{2} - 4x,\ x\in I \quad \text{and} \quad -\frac{1}{2} - 4x \le 7\frac{1}{2},\ x\in I$$

$$\Rightarrow \quad 4x < -\frac{1}{2} + 8\frac{1}{2} \quad \Big| \quad -4x \le 7\frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow \quad 4x < 8 \quad \Big| \quad -4x \le 8$$

$$\Rightarrow \quad x < 2,\ x\in I \quad \Big| \quad x \ge -2,\ x\in I$$

$$x < 2,\ x \ge -2 \text{ and } x\in I \quad i.e.\ -2 \le x < 2,\ x\in I$$

$\Rightarrow$ **Solution set : {–2, –1, 0 1}** **Ans.**

$\therefore$ **Solution set on a number line**

SECTION B (40 Marks)

*(Answer any **four** questions from this Section)*

***Question* 5 :**

(a) Given matrix B = $\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$. Find the matrix X if, X = $B^2 - 4B$. Hence, solve for *a* and *b* given X$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$. **[4]**

Solution :

$$X = B^2 - 4B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix} = \begin{bmatrix} \mathbf{5} & \mathbf{0} \\ \mathbf{0} & \mathbf{5} \end{bmatrix}$$ **Ans.**

$$X\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

$\Rightarrow \quad 5a = 5$ and $5b = 50$ *i.e.* $\boldsymbol{a = 1}$ and $\boldsymbol{b = 10}$ **Ans.**

(b) How much should a man invest in ₹ 50 shares selling at ₹ 60 to obtain an income of ₹ 450, if the rate of dividend declared is 10%. Also find his yield percent, to the nearest whole number. **[3]**

Solution :

Income on each share = 10% of ₹ 50 = ₹ 5

$\because$ Tota income = ₹ 450

$\therefore$ Total number of shares bought = $\dfrac{₹\,450}{₹\,5} = 90$

The man should invest = 90 × ₹ 60 = **₹ 5,400** **Ans.**

Let yield percent = $x\%$

$\therefore$ $x\%$ of ₹ 60 = 10% of ₹ 50

$\Rightarrow \quad \dfrac{x \times 60}{100} = \dfrac{10 \times 50}{100}$ *i.e.* $x = 8{\cdot}33$

$\therefore$ **Yield percent to the nearest whole number = 8%.** **Ans.**

(c) Sixteen cards are labelled as *a, b, c* ...., *m, n, o, p*. They are put in a box and shuffled. A boy is asked to draw a card from the box. What is the probability that the card drawn is :

(i) a vowel.

(ii) a consonant.

(iii) none of the letters of the word median. **[3]**

Solution :

(i) Since, vowels are *a, e, i* and *o* which are 4 in number.

$\therefore$ **P (a vowel)** = $\dfrac{4}{16} = \mathbf{\dfrac{1}{4}}$ **Ans.**

(ii) Since, number of consonants = Number of all the given letters – number of vowels in them = 16 – 4 = 12

$\therefore$ **P (a consonant)** = $\dfrac{12}{16} = \mathbf{\dfrac{3}{4}}$ **Ans.**

(iii) $\because$ Number of letters in the word median = 6

$\Rightarrow$ Number of letters not in the word median = 16 – 6 = 10

$\Rightarrow$ **P (none of the letters of the word median)** = $\dfrac{10}{16} = \mathbf{\dfrac{3}{8}}$ **Ans.**

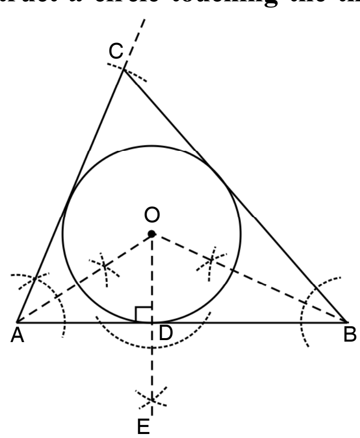
***Question* 6 :**

(a) Using a ruler and a compass construct a triangle ABC in which AB = 7 cm, ∠CAB = 60° and AC = 5 cm. Construct the locus of :

(i) points equidistant from AB and AC.

(ii) points equidistant from BA and BC.

Hence construct a circle touching the three sides of the triangle internally. **[4]**



Solution :

Construct triangle ABC with AB = 7 cm, $\angle CAB = 60°$ and AC = 5 cm.

(i) Draw the bisector of angle BAC to get the locus of points equidistant from AB and AC.

(ii) Draw the bisector of angle ABC to get the locus of points equidistant from BA and BC.

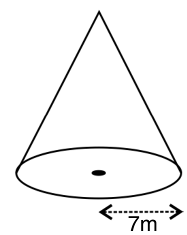
Let the two loci intersect each other at point O. From O, draw perpendicular to side AB which meets AB at point D.

Now taking O as centre and OD as radius draw a circle that will touch all the three sides of the given triangle internally.

Hence the required construction.

(b) A conical tent is to accommodate 77 persons. Each person must have 16m^3 of air to breathe. Given the radius of the tent as 7*m*, find the height of the tent and also its curved surface area. [3]

Solution :



Capacity of the conical tent = $77 \times 16 \text{ m}^3$

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 77 \times 16$$

$$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h = 77 \times 16$$

$$\Rightarrow \quad h = 24 \text{ m}$$

$\therefore$ **Height of the tent = 24 m** **Ans.**

Since, **stant height** $(l) = \sqrt{h^2 + r^2}$

$$= \sqrt{24^2 + 7^2} = 25 \text{ m}$$

$\therefore$ **Its curved surface area** $= \pi r l$

$$= \frac{22}{7} \times 7 \times 25 \text{ m}^2$$

$$= 550 \text{ m}^2$$ **Ans.**

(c) If $\frac{7m + 2n}{7m - 2n} = \frac{5}{3}$, use properties of proportion to find :

(i) $m : n$ **(ii) $\frac{m^2 + n^2}{m^2 - n^2}$** **[3]**

Solution :

(i) $\frac{7m + 2n}{7m - 2n} = \frac{5}{3}$

On applying componendo and dividendo, we get :

$$\frac{7m+2n+7m-2n}{7m+2n-7m+2n} = \frac{5+3}{5-3}$$

$$\Rightarrow \quad \frac{14m}{4n} = \frac{8}{2} \Rightarrow \frac{m}{n} = \frac{8}{2} \times \frac{4}{14} = \frac{8}{7} \Rightarrow \boldsymbol{m : n = 8 : 7} \quad \textbf{Ans.}$$

(ii) $$\frac{m}{n} = \frac{8}{7} \Rightarrow \frac{m^2}{n^2} = \frac{64}{49}$$

$$\Rightarrow \frac{\boldsymbol{m^2+n^2}}{\boldsymbol{m^2-n^2}} = \frac{64+49}{64-49} \text{ [Applying componendo and dividendo]}$$

$$= \frac{\mathbf{113}}{\mathbf{15}} \quad \textbf{Ans.}$$

Question **7 :**

(a) A page from the savings bank account is given below :

Date	Particulars	Amount Withdrawn (₹)	Amount Deposited (₹)	Balance (₹)
Jan. 7, 2016	B/F	–	–	3,000
Jan. 10, 2016	By cheque	–	2600	5600
Feb. 8, 2016	By self	1500	–	4100
Apr. 6, 2016	To cheque	2100	–	2000
May 4, 2016	By cash	–	6500	8500
May 27, 2016	By cheque	–	1500	10000

(i) Calculate the interest for the 6 months from January to June 2016, at 6% per annum.

(ii) If the account is closed on 1st July 2016, find the amount received by the account holder.

[Not in ICSE course now] [5]

(b) Use a graph paper for this question (Take 2 cm = 1 unit on both *x* and *y* axes)

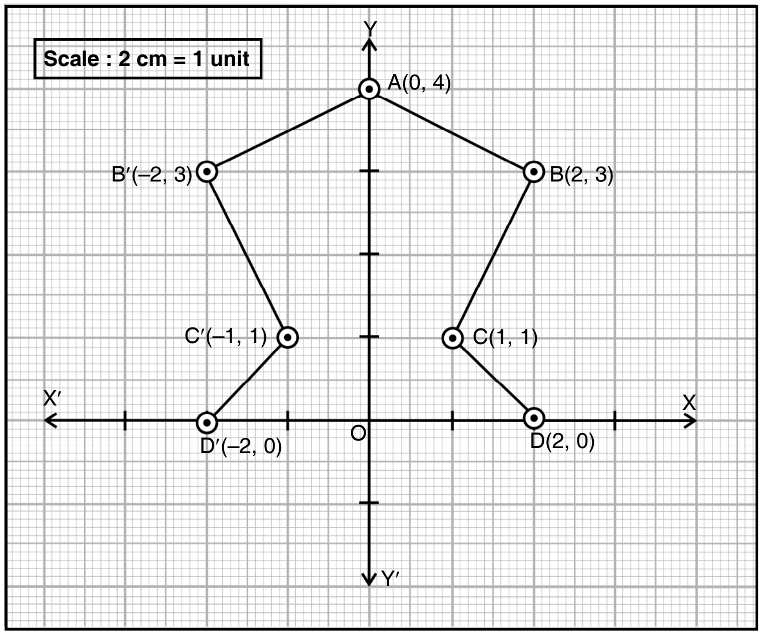
(i) Plot the following points :

A(0, 4), B(2, 3), C(1, 1) and D(2, 0).

(ii) Reflect points B, C, D on the y-axis and write down their coordinates. Name the images as B′, C′, D′ respectively.

(iii) Join the points A, B, C, D, D′, C′, B′ and A in order, so as to form a closed figure. Write down the equation of the line of symmetry of the figure formed. [5]

Solution :



The required line of symmetry is y-axis and its equation is $x = 0$.

***Question* 8 :**

(a) Calculate the mean of the following distribution using step deviation method.

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**
No. of students	**10**	**9**	**25**	**30**	**16**	**10**

[4]

Solution :

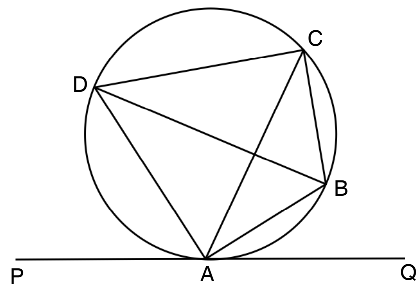
C.I.	**(f)**	**Mid-value (x)**	**Let A = 35 $d = x - A$**	$t = \frac{x - A}{i}$	$f \times t$
0-10	10	5	–30	–3	–30
10-20	9	15	–20	–2	–18
20-30	25	25	–10	–1	–25
30-40	30	35	0	0	0
40-50	16	45	10	1	16
50-60	10	55	20	2	20
	$\Sigma f = 100$				$\Sigma ft = -37$

$\therefore$ **Mean** $= A + \dfrac{\sum ft}{\sum f} \times i = 35 + \dfrac{-37}{100} \times 10 = \mathbf{31{\cdot}3}$ **Ans.**

(b) In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of $\angle CAQ$ and $\angle PAC$. If $\angle BAQ = 30°$, prove that :

(i) BD is diameter of the circle.

(ii) ABC is an isosceles triangle. [3]



Solution :

(i) AB is bisector of angle CAQ

$\Rightarrow \quad \angle CAB = \angle BAQ = 30°$

$\angle PAC = \angle PAQ - \angle CAQ = 180° - (30° + 30°) = 120°$

and AD bisects angle PAC

$$\Rightarrow \quad \angle PAD = \angle DAC = \frac{120°}{2} = 60°$$

Now, $\quad \angle DAB = \angle DAC + \angle CAB = 60° + 30° = 90°$

$\therefore$ $\angle DAB$ is angle in a semi-circle and so **BD is a diameter of the circle**.

(ii) $\because$ AB bisects angle CAQ, $\angle CAB = \angle BAQ = 30°$

$\because$ Angle of alternate segments are equal

$\angle ACB = \angle BAQ = 30°$

$\Rightarrow \quad \angle CAB = \angle ACB$

$\Rightarrow$ **ABC is an isosceles triangle. Ans.**

(c) The printed price of an air conditioner is ₹ 45,000/-. The wholesaler allows a discount of 10% to the shopkeeper. The shopkeeper sells the article to the customer at a discount of 5% of the marked price. Sales tax (under VAT) is charged at the rate of 12% at every stage. Find :

(i) VAT paid by the shopkeeper to the government.

(ii) The total amount paid by the customer inclusive of tax. [3]

Solution :

S.P. for the wholesaler = ₹ 45,000/- – 10% of ₹ 45,000/-

= ₹ 40,500

$\Rightarrow$ C.P. for shopkeeper = ₹ 40,500

S.P. for shopkeeper = ₹ 45,000 – 5% of ₹ 45,000

= ₹ 42,750

$\Rightarrow$ C.P. for customer = ₹ 42,750

(i) **VAT paid by the shopkeeper to the government**

$$= \text{Tax charged} - \text{Tax paid}$$

$$= 12\% \text{ of } ₹\ 42{,}750 - 12\% \text{ of } ₹\ 40{,}500$$

$$= ₹\ 5{,}130 - ₹\ 4{,}860 = \mathbf{₹\ 270}$$ **Ans.**

(ii) **The total amount paid by the customer inclusive of tax**

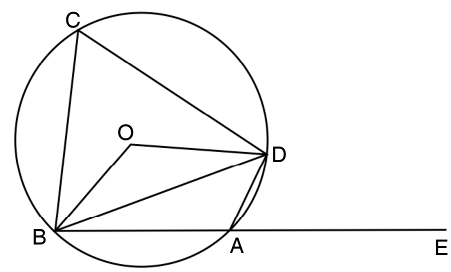
$$= \text{C.P. for customer} + \text{tax}$$

$$= ₹\ 42{,}750 + 12\% \text{ of } ₹\ 42{,}750$$

$$= \mathbf{₹\ 47{,}880}$$ **Ans.**

***Question* 9 :**

(a) In the figure given, O is the centre of the circle. ∠DAE = 70°. Find giving suitable reasons, the measure of



(i) ∠BCD

(ii) ∠BOD

(iii) ∠OBD **[4]**

Solution :

(i) **∠BCD** = Angle of alternate opposite segment

= ∠DAE = **70°** **Ans.**

(ii) ∵ Angle at centre is equal to twice the angle at remaining circumference

⇒ **∠BOD** = 2 × ∠BCD

= 2 × 70° = **140°** **Ans.**

(iii) In triangle OBD,

OB = OD = radius of the circle and ∠BOD = 140°

$$\therefore\ \angle \mathbf{OBD} = \frac{180° - 140°}{2} = \mathbf{20°}$$ **Ans.**

(b) A(–1, 3), B(4, 2) and C(3, –2) are the vertices of a triangle.

(i) Find the coordinates of the centroid G of the triangle.

(ii) Find the equation fo the line through G and parallel to AC. **[3]**

Solution :

(i) $$\textbf{Centroid G} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{-1+4+3}{3}, \frac{3+2-2}{3}\right) = \mathbf{(2,\ 1)}$$ **Ans.**

(ii) Slope of AC $= \dfrac{-2-3}{3+1} = \dfrac{-5}{4}$ and G = (2, 1)

⇒ The equation of the line through G and parallel to AC is :

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - 1 = -\frac{5}{4}(x - 2)$$

$$\Rightarrow \quad 4y - 4 = -5x + 10$$

$$\Rightarrow \quad \mathbf{5x + 4y = 14}$$ **Ans.**

(c) Prove that : $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta.$ **[3]**

Solution :

$$\textbf{L.H.S.} = \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta[2(1 - \sin^2\theta) - 1]}$$

$$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2 - 2\sin^2\theta - 1)}$$

$$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(1 - 2\sin^2\theta)} = \frac{\sin\theta}{\cos\theta} = \tan\theta = \textbf{R.H.S.}$$

***Question* 10 :**

(a) The sum of the ages of Vivek and his younger brother Amit is 47 years. The product of their ages in years is 550. Find their ages. **[4]**

Solution :

Let the age of Vivek = x years $\Rightarrow$ Age of Amit = $(47 - x)$ years

Given : $x(47 - x) = 550$ $\Rightarrow$ $47x - x^2 = 550$

i.e. $x^2 - 47x + 550 = 0$ $\Rightarrow$ $x^2 - 25x - 22x + 550 = 0$

i.e. $x(x - 25) - 22(x - 25) = 0$ $\Rightarrow$ $(x - 25)(x - 22) = 0$

i.e. $x - 25 = 0$ or $x - 22 = 0$ $\Rightarrow$ $x = 25$ or $x = 22$

Since, Amit is younger than Vivek

$\therefore$ **Age of Vivek = 25 years and age of Amit = 22 years** **Ans.**

(b) The daily wages of 80 workers in a project are given below.

Wages (in ₹)	**400-450**	**450-500**	**500-550**	**550-600**	**600-650**	**650-700**	**700-750**
No. of workers	**2**	**6**	**12**	**18**	**24**	**13**	**5**

Use a graph paper to draw an ogive for the above distribution. (Use a scale of 2 cm = ₹ 50 on x-axis and 2 cm = 10 workers on y-axis). Use your ogive to estimate :

(i) the median wages of the workers.

(ii) the lower quartile wage of workers.

(iii) the number of workers who earn more than ₹ 625 daily. **[6]**

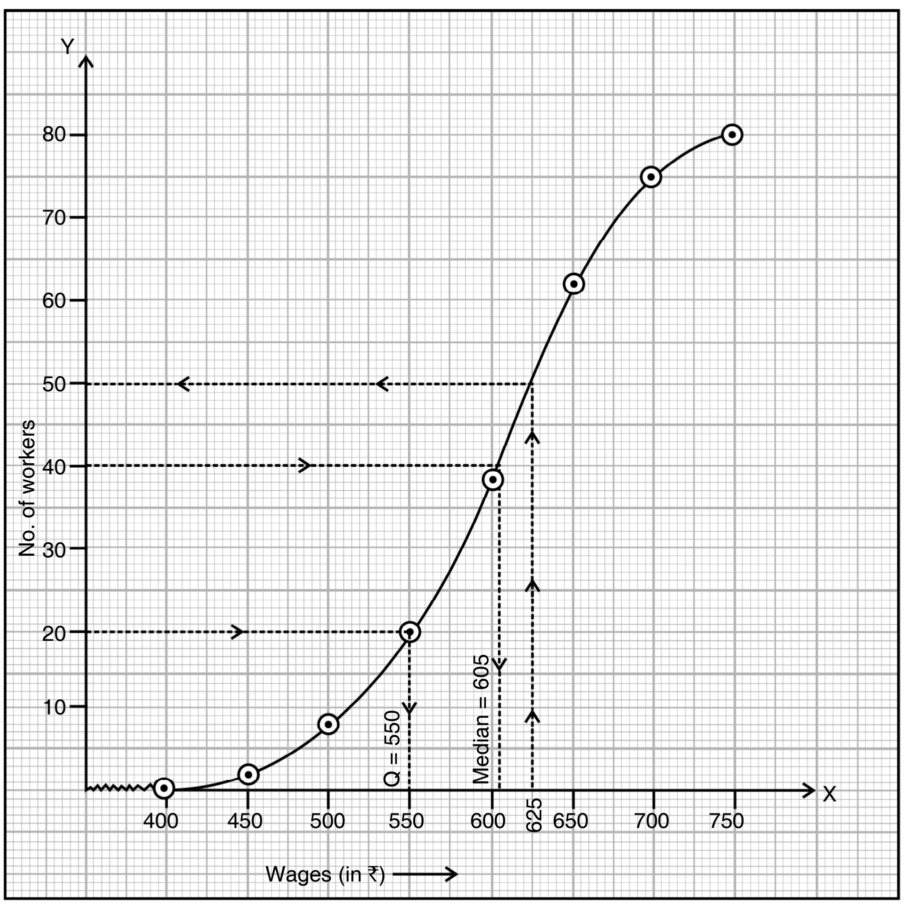
Solution :

Wages (in ₹)	**No. of Workers (f)**	**Cummulative frequency (C.f.)**
400-450	2	2
450-500	6	8
500-550	12	20
550-600	18	38
600-650	24	62
650-700	13	75
700-750	5	80

On the graph paper, drawn to given scale, mark the points (450, 2), (500, 8), (550, 20), (600, 38), (650, 62), (700, 75) and (750, 80). Then draw a free-hand curve passing through the points marked, starting from the lower limit of the first class and terminating at upper limit of the last class. The curve (graph) so obtained is an ogive, as shown below :

(i) **Median** $= \left(\dfrac{n}{2}\right)^{\text{th}}$ term

$= \left(\dfrac{80}{2}\right)^{\text{th}}$ term $= 40^{\text{th}}$ term $=$ **₹ 605** **Ans.**



(ii) **Lower quartile** = $\left(\frac{n}{4}\right)^{\text{th}}$ term

$$= \left(\frac{80}{4}\right)^{\text{th}} \text{ term} = 20^{\text{th}} \text{ term} = ₹\ \mathbf{550} \qquad \textbf{Ans.}$$

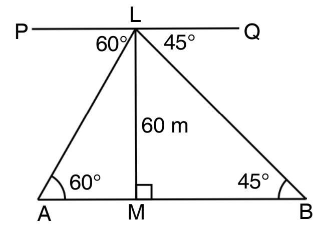
(iii) Since, number of workers who earn upto ₹ 625 = 50

∴ The number of workers who earn more than ₹ 625 = 80 – 50 = **30** **Ans.**

***Question* 11 :**

(a) The angles of depression of two ships A and B as observed from the top of a light house 60 m high are 60° and 45° respectively. If the two ships are on the opposite sides of the light house, find the distance between the two ships. Give your answer correct to the nearest whole number. [4]

Solution :



Let LM be the light house such that LM = 60 m

A and B are two ships which are on opposite sides of light house LM.

Angles of depression of the two ships from the top of light house are 60° and 45° respectively.

According to the diagram :

⇒ ∠PLA = ∠LAM = 60° and ∠QLB = ∠LBM = 45°

In ΔLAM, $\tan 60° = \frac{60 \text{ m}}{\text{AM}} \Rightarrow \sqrt{3} = \frac{60 \text{ m}}{\text{AM}}$ and $\text{AM} = \frac{60}{\sqrt{3}}$ m

$$= \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \text{ m} = 20\sqrt{3} \text{ m} = 34{\cdot}64 \text{ m}$$

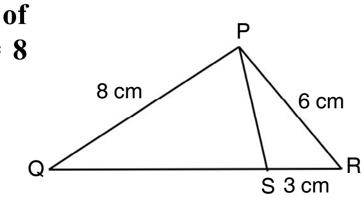
In ΔLBM, $\tan 45° = \frac{60 \text{ m}}{\text{BM}} \Rightarrow 1 = \frac{60 \text{ m}}{\text{BM}}$ and BM = 60 m

∴ **The distance btween the two ships** = AM + BM

= 34·64 m + 60 m

= 94·64 m = **95 m** **Ans.**

(b) PQR is a triangle. S is a point on the side QR of ΔPQR such that ∠PSR = ∠QPR. Given QP = 8 cm, PR = 6 cm and SR = 3 cm.



(i) Prove ΔPQR ~ ΔSPR.

(ii) Find the lengths of QR and PS.

(iii) $\frac{\textbf{area of } \Delta \textbf{PQR}}{\textbf{area of } \Delta \textbf{SPR}}$ [3]

Solution :

(i) In ΔPQR and ΔSPR,

∠QPR = ∠PSR (Given)

∠R = ∠R (Common)

⇒ ΔPQR ~ ΔSPR (by A.A.)

Hence Proved.

(ii) $\Delta PQR \sim \Delta SPR$

$$\Rightarrow \quad \frac{QR}{PR} = \frac{PQ}{PS} = \frac{PR}{SR}$$

$$\Rightarrow \quad \frac{QR}{6\text{ cm}} = \frac{8\text{ cm}}{PS} = \frac{6\text{ cm}}{3\text{ cm}}$$

$$\Rightarrow \quad \frac{QR}{6\text{ cm}} = \frac{6\text{ cm}}{3\text{ cm}} \text{ and } \frac{8\text{ cm}}{PS} = \frac{6\text{ cm}}{3\text{ cm}}$$

$\Rightarrow$ **QR = 12 cm** and **PS = 4 cm** **Ans.**

(iii) ΔPQR is similar to ΔSPR

$$\Rightarrow \quad \frac{\textbf{Area of } \Delta\textbf{PQR}}{\textbf{Area of } \Delta\textbf{SPR}} = \frac{PR^2}{SR^2} \qquad \text{(Given)}$$

$$= \frac{(6\text{ cm})^2}{(3\text{ cm})^2} = \frac{4}{1} = \mathbf{4 : 1}$$ **Ans.**

(c) Mr. Richard has a recurring deposit account in a bank for 3 years at 7·5% p.a. simple interest. If he gets ₹ 8325 as interest at the time of maturity, find :

(i) The monthly deposit

(ii) The maturity value. **[3]**

Solution :

(i) $$I = \frac{P \times n \times (n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow \quad ₹\ 8325 = \frac{P \times 36 \times 37}{2 \times 12} \times \frac{7{\cdot}5}{100} \qquad [\because n = 3 \text{ years} = 36 \text{ months}]$$

$$\Rightarrow \quad P = \frac{₹\ 8325 \times 2 \times 12 \times 100}{36 \times 37 \times 7{\cdot}5} = ₹\ 2000$$

$\therefore$ **The monthly deposit = ₹ 2000** **Ans.**

(ii) **The maturity value** = $P \times n + I$

= ₹ 2000 × 36 + ₹ 8325

= **₹ 80325** **Ans.**